

A new method for a global fit of the CKM matrix

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We report on a new method to a global fit of the CKM matrix by using the necessary and sufficient condition the data have to satisfy in order to find a unitary matrix compatible with them. This condition writes as $-1 \leq \cos \varphi \leq 1$ where φ is the phase that accounts for CP violation. By using it we get that the experimental data are to a high degree compatible to unitarity and that φ takes values around 90° , in contrast to the previous determinations. Numerical results are provided for the CKM matrix entries, the mixing angles between generations and all the angles of the standard unitarity triangle.

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The determination of the Cabibbo-Kobayashi-Maskawa matrix that parameterizes the weak charged current interactions of quarks is a lively subject in particle physics. The four independent parameters of this matrix govern all flavor changing transitions of quarks in the Standard Model, and their determination is an important task for both experimenters and theorists. The large interest in the subject is also reflected by the workshops organized in the last two years whose main subject was the CKM matrix [1, 2, 3].

Although it seems that at the level of experts there exists a consensus concerning the method of extracting from experimental data information about the CP violating phase [1]-[6], the usual method of considering only the orthogonality of the first and third columns is not reliable as we shall show in the paper. The main shortcomings of the current approach are the following: a) one uses one unitarity triangle instead of six; b) the currently used four independent parameters are not re-phasing invariant; c) one works with an approximation of the standard parameterization which may lead to inconsistencies.

The aim of this paper is to propose an alternative method for imposing unitarity by fully exploiting the constraints implied by it. We consider that the second-generation B-decay experiments of the LHC era, when the accuracy will make a tremendous difference, require to keep improving all the tools we are working with. Our approach provides the necessary and sufficient condition the data have to satisfy in order to find a unitary matrix compatible with them; this condition is given by $-1 \leq \cos \varphi \leq 1$, where φ is the phase entering the CKM matrix. We construct a theoretical model and we use it to test the unitarity property of the data as they are provided by the Particle Data Group (PDG) [7]. We find that one can reconstruct from PDG data a unitary matrix and the phase φ is close to $\pi/2$.

Our theoretical framework is as follows. We use the

standard parameterization advocated by PDG that we write it in a completely rephasing invariant form as

$$U_{CKM} = \quad (1)$$

$$\begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\varphi} - c_{12}s_{23}s_{13} & c_{12}c_{23}e^{i\varphi} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23}e^{i\varphi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\varphi} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ for the generation labels $ij = 12, 13, 23$, and φ is the phase that encodes the breaking of the CP -invariance. It is easily seen that by multiplying a unitary matrix at left and at right with arbitrary diagonal phase matrices, the phase invariance implies that we can freely choose the entries of one column and of one row as non-negative numbers. We used this property for rewriting the standard parameterization [8] in the above form. We remind that phase invariance of the U_{CKM} -transformed quark wave functions is a requirement for physically meaningful quantities.

On the other hand from experiments one measures a matrix whose entries are positive

$$V = \begin{pmatrix} V_{ud}^2 & V_{us}^2 & V_{ub}^2 \\ V_{cd}^2 & V_{cs}^2 & V_{cb}^2 \\ V_{td}^2 & V_{ts}^2 & V_{tb}^2 \end{pmatrix} \quad (2)$$

entries that, in principle, can be determined from the weak decays of the relevant quarks, and/or from deep inelastic neutrino scattering [7]. In other words we make a clear distinction between the unitary CKM matrix U_{CKM} and the *positive* matrix V provided by the data. The main theoretical problem is to see if from a matrix as (2) one can reconstruct a unitary matrix as (1). If the experimental data are compatible with unitarity the weakest form of this property is expressed as follows

$$\begin{aligned} \sum_{i=d,s,b} V_{ji}^2 - 1 &= 0, \quad j = u, c, t \\ \sum_{i=u,c,t} V_{ij}^2 - 1 &= 0, \quad j = d, s, b \end{aligned} \quad (3)$$

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We stress that the above relations does not test the unitarity, as it is usually stated in many papers; they are necessary but not sufficient conditions. The class of positive matrices satisfying Eqs. (3) is considerable larger than the class of positive matrices coming from unitary matrices. The set (3) is known in the mathematical literature as doubly stochastic matrices, and the subset coming from unitary matrices $V_{ij}^2 = |U_{ij}|^2$ is known asunistochastic ones [9]. The double stochastic matrices have an important property, they are a convex set, i.e. if V_1 and V_2 are doubly stochastic so is their convex combination $\alpha V_1 + (1 - \alpha)V_2$, $\alpha \in [0, 1]$ as it is easily checked.

The first problem to solve is to find a necessary and sufficient criterion for discrimination between the two sets. Assuming the experimental data (2) come from a unitary matrix as (1) we obtain the following relations between the parameters entering U_{CKM} and V matrices

$$\begin{aligned} V_{ud}^2 &= c_{12}^2 c_{13}^2, \quad V_{us}^2 = s_{12}^2 c_{13}^2, \quad V_{ub}^2 = s_{13}^2, \\ V_{cb}^2 &= s_{23}^2 c_{13}^2, \quad V_{tb}^2 = c_{13}^2 c_{23}^2, \\ V_{cd}^2 &= s_{12}^2 c_{23}^2 + s_{13}^2 s_{23}^2 c_{12}^2 + 2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\varphi \\ V_{cs}^2 &= c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\varphi \\ V_{td}^2 &= s_{13}^2 c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 - 2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\varphi \\ V_{ts}^2 &= s_{12}^2 s_{13}^2 c_{23}^2 + c_{12}^2 s_{23}^2 + 2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\varphi \end{aligned} \quad (4)$$

It is easily seen that the parameterization (4) satisfies identically the relations (3), and also that CP -violation requires

$$\theta_{ij} \neq 0, \quad ij = 12, 13, 23, \quad \theta_{12} \neq \pi/2, \quad \text{and} \quad \theta_{23} \neq \pi/2 \quad (5)$$

We deduce from the factors that multiply $\cos\varphi$ in Eqs.(4) that an entire region around 0 and $\pi/2$ is forbidden for the above parameters. Using the numerical values from PDG data one can get values for $\cos\varphi$ outside the “physical” range $[-1, 1]$, or even complex as we will show in

the following. The above relations provides the necessary and sufficient condition the data have to satisfy in order the matrix (2) comes from a unitary matrix, and this condition is

$$-1 \leq \cos\varphi \leq 1 \quad (6)$$

Since in relations (4) φ enters only in the cosine function we can take $\varphi \in [0, \pi]$ without loss of generality.

The last four relations (4) provide us formulas for $\cos\varphi$ and these formulas have to give the same number when comparing theory with experiment, by supposing the data come from a unitary matrix. Their explicit form depends on the independent four parameters we choose to parameterize the data, and we will always choose these parameters as four experimentally measurable quantities, i.e. V_{ij}^2 . An other reason is that $V_{ij}^2 = |U_{CKM}^{ij}|^2$ being square of the moduli functions they are repahising invariant; the CP violating phase φ does not share this property [10]-[11]. By consequence the parameters entering on the right hand side of the relation

$$s_{13} e^{-i\delta} = A \lambda^3 (\rho - i\eta)$$

usually used by the physics community [6, 12] are *not* repahising invariant because the left hand side is not. Another invariant parameter is the Jarlskog invariant J [13], but it is not a measurable quantity. Depending on the explicit choice of the four independent parameters we get one, two, three or four different expressions for $\cos\varphi$; e.g. if we take V_{ud} , V_{us} , V_{cd} , V_{cs} , as independent parameters we get

$$\begin{aligned} s_{12} &= \frac{V_{us}}{\sqrt{V_{ud}^2 + V_{us}^2}}, \quad s_{13} = \sqrt{1 - V_{cd}^2 - V_{cs}^2}, \quad \text{and} \\ s_{23} &= \frac{\sqrt{1 - V_{cd}^2 - V_{cs}^2}}{\sqrt{V_{ud}^2 + V_{us}^2}} \end{aligned} \quad (7)$$

and from the sixth Eqs.(4) we have

$$\cos\varphi = \frac{V_{ud}^2 + V_{cd}^2 V_{ud}^2 + V_{cs}^2 V_{ud}^2 + V_{ud}^4 - V_{cs}^2 V_{ud}^4 + V_{us}^2 - V_{cd}^2 V_{us}^2 - V_{cs}^2 V_{us}^2 + V_{cd}^2 V_{ud}^2 V_{us}^2 - V_{us}^4 - V_{cs}^2 V_{ud}^2 V_{us}^2 + V_{cd}^2 V_{us}^4}{2 V_{ud} V_{us} \sqrt{1 - V_{cd}^2 - V_{cs}^2} \sqrt{1 - V_{ud}^2 - V_{us}^2} \sqrt{V_{ud}^2 + V_{us}^2 + V_{cd}^2 + V_{cs}^2} - 1} \quad (8)$$

In this case, other two independent formulas are given by the last two equations in relations (4), and they have to give (almost) identical numerical results if the data are compatible with unitarity. In general the data will give different numerical values for the three func-

tions expressing $\cos\varphi$. If the independent parameters are V_{ud} , V_{ub} , V_{cd} , V_{cb} , i.e. we use the information contained in the first and the third columns, we obtain four different expressions for $\cos\varphi$, and in this case the mixing angles θ_{ij} are given by

$$s_{12} = \frac{\sqrt{1 - V_{ud}^2 - V_{ub}^2}}{\sqrt{1 - V_{ub}^2}}, \quad s_{13} = V_{ub},$$

$$s_{23} = \frac{V_{cb}}{\sqrt{1 - V_{ub}^2}} \quad (9)$$

Looking at Eqs. (7) and (9) we see that the expressions defining the mixing angles are quite different. Thus if the data are compatible to the existence of a unitary matrix these mixing angles have to be equal, and this is a necessary condition for unitarity. As a warning what we said before can be summarized as follows: the unitarity property is a property of *all the CKM matrix elements* and not the property of a row and/or a column, as it is considered by many people working in the field.

To better understand the above considerations, let us consider the toy model defined by $V_{ij}^2 = 1/3$. It is easily seen that no matter how we choose the independent variables we get the same numerical results for s_{ij} when we use the *exact values* $V_{ij}^2 = 1/3$. In this case we have

$$s_{12} = s_{23} = \frac{1}{\sqrt{2}}, \quad s_{13} = \frac{1}{\sqrt{3}}$$

and from any of the last four equations (4) we get $\cos \varphi = 0$, i.e. $\varphi = \pi/2$. Then from Eq.(1) we obtain the unitary matrix

$$U_{toy} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ e^{2\pi i/3} & e^{4\pi i/3} & 1 \\ e^{4\pi i/3} & e^{2\pi i/3} & 1 \end{pmatrix} \quad (10)$$

which is the matrix of the finite Fourier transform in $d = 3$ dimensions. The above matrix has the property that it maximizes the Jarlskog invariant $J = s_{12}s_{13}s_{23}c_{12}c_{13}^2c_{23} \sin \varphi$, $J_{max} = 1/(6\sqrt{3})$. To see that the orthogonality properties of columns and rows are not trivial, let us suppose that the second column of the U_{toy} matrix has the form $(1, 1, 1)^t$, where t means transpose. It is easily seen that the orthogonality of the first column with the second and third ones of this new matrix are exactly satisfied, but the second and the third columns *are not orthogonal*! The same happens with all the rows, i.e. the orthogonality property is very strong implying supplementary constraints that until now were never been used at their maximum potential. In the real case of the experimental data the situation is worse since we use approximate values for V_{ij} and then the values of s_{ij} depend on the choice of the independent parameters that define the model and are no more independent of them as in the case of the U_{toy} matrix.

Now we define a test function that should take into account the double stochasticity property expressed by the conditions (3) and the fact that in general the numerical values of data are such that $\cos \varphi$ depends on the choice

of the four independent parameters. Our proposal is

$$\chi_1^2 = \sum_{i < j} (\cos \varphi^{(i)} - \cos \varphi^{(j)})^2 + \sum_{j=u,c,t} \left(\sum_{i=d,s,b} V_{ji}^2 - 1 \right)^2$$

$$+ \sum_{j=d,s,b} \left(\sum_{i=u,c,t} V_{ij}^2 - 1 \right)^2 \quad (11)$$

Testing our model (11) on the toy matrix (10) we found that by adding a second component of the form

$$\chi_2^2 = \sum_{i=u,c} \sum_{j=d,s,b} \left(\frac{V_{ij} - \tilde{V}_{ij}}{\sigma_{ij}} \right)^2 \quad (12)$$

the results improve, where \tilde{V}_{ij} is a numerical matrix that describes the experimental data, and σ is the matrix of errors associated to \tilde{V}_{ij} . The test has also shown that better results are obtained when the number of $\cos \varphi$ is large. In the last sum we use only the data coming from the first two rows because the entries of the third row are not yet measured.

The above expressions will be used to test globally the unitarity property of the experimental data.

In the following we test our method on the published data, i.e. we want to see if our necessary and sufficient criterion, $-1 \leq \cos \varphi \leq 1$, could constrain enough the data. For that we use the PDG [7] data, the fit [5] and its recent up-to-date results [6], and consider that a good starting point for a comparison of the unitarity triangle approach, almost exclusively used nowadays, with our method is to look at the central values from the above cited papers. These values are given in relation (13) where we used the notation \sqrt{V} to denote the numerical CKM data matrix as it is usually provided. All matrices (13) satisfy quite well the stochasticity property (3) as it is seen from the Table 1.

$$\sqrt{V_{PDG}} = \begin{pmatrix} 0.97485 & 0.2225 & 0.00365 \\ 0.2225 & 0.974 & 0.041 \\ 0.009 & 0.0405 & 0.99915 \end{pmatrix}$$

$$\sqrt{V_{CKMfG}[5]} = \begin{pmatrix} 0.97504 & 0.2221 & 0.0035 \\ 0.2220 & 0.97422 & 0.0408 \\ 0.0079 & 0.04025 & 0.99917 \end{pmatrix} \quad (13)$$

$$\sqrt{V_{CKMfG}[6]} = \begin{pmatrix} 0.97400 & 0.2265 & 0.00387 \\ 0.2264 & 0.97317 & 0.04113 \\ 0.00826 & 0.04047 & 0.999146 \end{pmatrix}$$

Here CKMfG denotes the CKM fitter Group.

As we said before $\cos \varphi$ depends on the four independent parameters we use to obtain it and for comparison we took six groups of independent parameters in the Table 2. It is easily seen that the central values from all V_{PDG} and $CKMfG$ are not compatible with

	PDG [7]	HLLL [5]	HLLL [6]
r_1	-1.48×10^{-4}	4.37×10^{-5}	-6.77×10^{-6}
r_2	-1.37×10^{-4}	5.32×10^{-5}	8.49×10^{-6}
r_3	2.2×10^{-5}	2.32×10^{-5}	-1.22×10^{-6}
c_1	-8.02×10^{-5}	4.94×10^{-5}	1.19×10^{-6}
c_2	-1.78×10^{-4}	5.31×10^{-5}	-8.02×10^{-8}
c_3	-4.96×10^{-6}	1.76×10^{-5}	-6.17×10^{-7}

TABLE I: r_i and c_i , $i = 1, 2, 3$, are the values on the left hand side in Eqs.(3) calculated using the data (13)

unitarity, although they are compatible with the double stochasticity property to an acceptable accuracy. More, the data show that $\cos \varphi$ can take even imaginary values and these values can not be properly processed in the usual approach of the unitarity triangle and Wolfenstein approximation, which are the starting points of all the present analysis of the CKM data. In all the cases it is s_{13} that causes the trouble. For the PDG data $s_{13} = \sqrt{V_{cs}^2 + V_{ts}^2 - V_{ud}^2} = 4.0 \times 10^{-3} i$ and, respectively, $s_{13} = \sqrt{V_{cd}^2 + V_{cs}^2 + V_{td}^2 + V_{ts}^2 - 1} = 10^{-2} i$; for the CKM fitter Group results [5] it has the form $s_{13} = \sqrt{1 - V_{ud}^2 - V_{us}^2} = 5.6 \times 10^{-3} i$. On the other hand such an incompatibility cannot, in principle, be detected by using the Wolfenstein parameterization, because quantities as $s_{13} = \sqrt{1 - V_{ud}^2 - V_{us}^2}$, that may become imaginary in some cases, are usually approximated by $s_{13} = V_{ub}$ and never appear in the usual approach. The conclusion is that unitarity requires a very fine tuning between all the entries of the matrix (2) and our method could put strong constraints on the CKM matrix.

For the proper fit we considered six groups of four independent parameters, those appearing in the first column of Table 2, that lead to 17 different expressions for $\cos \varphi$. We considered that these six groups could be equivalent to the six orthogonality relations implied by unitarity. For minimization of the χ^2 we used the *FindMinimum* function provided by *Mathematica*. Special care was taken for properly treating the cases that lead to $\cos \varphi$ values outside the physical range.

We started the fit by using the information provided by the first group of four parameters in Table 2 since we wanted a comparison with the unitarity triangle approach that uses the same information. With no independent parameters we found a $\chi^2 = \chi_1^2 + \chi_2^2 = 3.8 \times 10^{-3}$ and we used the V_{ij} determined parameters to test all the seventeen $\cos \varphi^{(i)}$. We found that nine values, for $i = 1, \dots, 4, 8, 13, \dots, 17$, are around 0.05, their mean being $\langle \cos \varphi \rangle = 0.0518$. However we found also a few discrepancies: $\cos \varphi^{(5)} = \cos \varphi^{(6)} = 0.635$ and $\cos \varphi^{(7)} = -0.02$; even worse four values for $i = 9, \dots, 12$ are outside the physical region. We interpret this phenomenon as showing that the use of only one orthogonality constraint leads to non reliable results.

Taking into account the $\cos \varphi^{(i)}$ provided by the sec-

Para's	$\cos \varphi$	PDG [7]	HLLL [5]	HLLL [6]
$V_{ud}, V_{ub},$ V_{cd}, V_{cb}	$\cos \varphi^{(1)}$	-1.00237	1.30599	0.483707
	$\cos \varphi^{(2)}$	1.10268	0.443955	0.362787
	$\cos \varphi^{(3)}$	0.232663	0.506068	0.466784
	$\cos \varphi^{(4)}$	0.647108	0.596443	0.458158
$V_{ud}, V_{us},$ V_{cd}, V_{cs}	$\cos \varphi^{(5)}$	0.348363	-0.376647 i	0.473185
	$\cos \varphi^{(6)}$	0.690208	0.130873 i	0.459106
	$\cos \varphi^{(7)}$	-0.407954	-0.92185 i	0.472234
$V_{cd}, V_{cs},$ V_{td}, V_{ts}	$\cos \varphi^{(8)}$	0.452312 i	0.577013	0.459978
$V_{ud}, V_{ub},$ V_{cs}, V_{tb}	$\cos \varphi^{(9)}$	1.02868	0.718312	0.354383
	$\cos \varphi^{(10)}$	-0.997118	1.29883	0.484069
	$\cos \varphi^{(11)}$	0.236042	0.49466	0.467150
	$\cos \varphi^{(12)}$	0.573777	0.87161	0.449737
$V_{ud}, V_{ub},$ V_{cs}, V_{td}	$\cos \varphi^{(13)}$	-0.943474	0.577013	0.479405
	$\cos \varphi^{(14)}$	0.27128	1.30443	0.462446
	$\cos \varphi^{(15)}$	-0.176829	0.503588	0.558023
$V_{ud}, V_{cd},$ V_{cs}, V_{ts}	$\cos \varphi^{(16)}$	1.15643 i	1.07607	0.473158
	$\cos \varphi^{(17)}$	0.082041 i	0.464743	0.459054

TABLE II: All the central data (13) coming from [7], [5] and [6] are not compatible with unitarity, although the last fit [6] is considerably better than the published one. The left column contains the six groups of independent parameters used in our analysis.

ond group of independent parameters the fit improves, all the $\cos \varphi^{(i)}$ being inside the physical region, and the maximum difference between cosines is of the order $5. \times 10^{-2}$ and φ is around 58° , value compatible to that provided by the CKM fitter Group [6]. Including now in χ^2 all the the first twelve $\cos \varphi^{(i)}$ we found that the results considerably improve, and the difference between $\cos \varphi^{(i)}$ is of the order $2. \times 10^{-4}$. The big surprise was that φ takes values close to 90° ! For comparison, with our method we obtained by using the up-to-date V_{ij} parameters provided by the CKM fitter Group [6] that $0.354383 \leq \cos \varphi^{(i)} \leq 0.558023$, values that are listed in the last column of Table 2, with a mean value $\langle \cos \varphi \rangle = 0.460198$ and $\sigma = 0.0436841$ which leads to a CP phase $\varphi = 62.0001^\circ$, the interval of variation being $59.7428^\circ \leq \varphi \leq 65.3853^\circ$, results that almost coincide with that given in [6] that are $50^\circ \leq \varphi \simeq \gamma \leq 72^\circ$. In our opinion these results show the limits for the prediction power of the unitarity triangle approach.

If we use only the values provided by the first group in Table 2, one gets $\varphi = 63.7^\circ \pm 3.^\circ$. The difference between the two fits comes from their quality. In the following by using our method we found that χ^2 takes values in the interval $2. \times 10^{-7} \leq \chi_1^2 \leq 1.7 \times 10^{-3}$, while the same expression provides $\chi_1^2 = 2.95$ when using the last V_{ij} values obtained by CKM fitter Group.

We included step by step all the constraints implied by the six groups and obtained three different matrices; they were used to obtain another one by using the convexity property of the unistochastic matrices. Other matrices

were obtained by relaxing the condition (12) by taking all combinations with five and respectively four V_{ij} that provided $\binom{6}{1} + \binom{6}{2} = 21$ new matrices. The set of these matrices was considered as 25 independent “experiments” on which the statistics was done. For $\cos\varphi$ these lead to $25 \times 17 = 425$ values that gave $\varphi = 89.9962^0 \pm 0.0767^0$. The interval for the χ^2_1 values for all the 25 matrices was shown above and the final results are shown in the Table 3. In fact the obtained values can be improved.

One way to do this is to consider the number of all the possible four independent parameters V_{ij} which is $\binom{9}{4} = 126$, but not all are independent; we have to exclude all groups of four parameters that contain all entries from a row or a column, whose number is 36, and also all groups that are not independent because of the unitarity, e.g. $(V_{ud}, V_{ub}, V_{cs}, V_{ts})$, whose number is 9. This leads to $126 - 36 - 9 = 81$ groups and the estimated number of $\cos\varphi$ is about 240. The only trouble is that the fitting procedure is very time consuming even on a good work station.

Looking at the final results we see that V_{ij} values are not far from those provided by other fits. The striking feature concerns the values for the angles of the unitarity triangle, α, β and γ , angles that are not obtained from the fit, but from parameters provided by the fit. One sees that with a very good approximation the standard unitarity triangle is almost a rectangle one. We have $\varphi > \gamma$ where their difference is about $4. \times 10^{-2}$. However this leads to the unexpected result, $\sin 2\alpha \simeq \sin 2\beta$, relation that is not satisfied by the experimental data, which in particular are not very clean. We obtain that $\sin 2\beta = 0.677$, value that is not far from the world average of the BaBar [15] and Belle [16] experiments, that is $\sin 2\beta = 0.736 \pm 0.049$ [14]. In fact both angles α and β are at the lower extremity of the typical ranges for them [14], $70^0 \leq \alpha \leq 130^0$, $20^0 \leq \beta \leq 30^0$. We mention that a rectangle unitarity triangle was found in [17] but with $\alpha = 90^0$.

With our values we find $Im(U_{CKMts}^* U_{CKMtd}) = (1.33 \pm 0.03) \times 10^{-4}$ which is compatible to that used in [18] for determination of the CP-violating ratio ϵ'/ϵ .

In conclusion we can say that the unitarity triangle results are not reliable and that our approach could outperform by far all the other methods used to reconstruct a CKM unitary matrix from experimental data.

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Quantity	Central value \pm error
V_{ud}	0.974868 ± 0.00002
V_{us}	0.222755 ± 0.00003
V_{ub}	$(3.59529 \pm 0.0021)10^{-3}$
V_{cd}	0.222568 ± 0.000021
V_{cs}	0.974049 ± 0.00003
V_{cb}	$(4.11362 \pm 0.00285)10^{-2}$
V_{td}	$(9.80945 \pm 0.00244)10^{-3}$
V_{ts}	$(4.01109 \pm 0.00264)10^{-2}$
V_{td}	0.999147 ± 0.000028
$\sin \theta_{12}$	0.222725 ± 0.0000455
$\sin \theta_{13}$	$(3.58334 \pm 0.306)10^{-3}$
$\sin \theta_{23}$	$(4.11351 \pm 0.03325)10^{-2}$
J	$(3.20 \pm 0.27)10^{-5}$
φ	$89.9962^0 \pm 0.0767^0$
α	$68.7517^0 \pm 0.02^0$
β	$21.2862^0 \pm 0.02^0$
γ	$89.9591^0 \pm 0.03^0$

TABLE III: Fit results and errors using the standard input from PDG data. The results show that there is a unitary matrix compatible with the data. The values for α, γ and φ strongly disagree with the previous determinations.

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